

Using Artificial Organisms To Study The Evolution of Backbones in Fish

C.W. Liew

Robert G. Root

John H. Long Jr.

T. J. Koob

Dept of Computer Science Dept of Mathematics Department of Biology University of South Florida

Lafayette College,

Lafayette College,

Vassar College,

Tampa, FL

Easton, PA 18042

Easton, PA 18042

Poughkeepsie, NY 12604

liew@cs.lafayette.edu

Megan A. Cummins

Lafayette College,

Easton, PA 18042

Abstract—

This paper describes the use of both robotic and digital organisms to help in the study and understanding of the evolution of biological structures. Our premise in this paper is that simulations using robotic and digital organisms are an effective methodology for studying how some features evolved in swimming fish. Experiments with the artificial organisms allow us to evaluate the hypothesis that backbones evolved in fish in part because they result in higher velocity, acceleration and maneuverability. The use of both robotic and digital organisms provides the ability to (1) use computers to efficiently explore a very large search space of possibilities, (2) validate (using the robotic organisms) that the digital models accurately reflect the physical constraints of the environment.

I. INTRODUCTION

EVOLUTIONARY biologist have found it difficult to research and determine the paths of evolution because of either missing or incomplete fossil records. Biologists can make predictions about possible links between species and also possible evolutionary pathways but without the fossils, it is difficult to determine which paths are more likely, let alone which is the path that was actually taken. In this paper, we describe how artificial (digital) life technology and evolutionary computing techniques were combined to help biologists determine plausible evolutionary directions and paths. We combine these elements evolving robotic and digital organisms to test predictions about the origin of vertebrae in the early history of vertebrates. Determining the paths along which physical features or structures evolved is a difficult problem for biologists. In most cases, the fossil record is missing all or most of the intermediate points (species) along the paths. Therefore many possible paths can be constructed but the fossil records do not help us discriminate and determine the actual or likely evolutionary paths.

We are studying the evolution of the backbone in cartilaginous fish, in particular we are trying to determine what advantages a segmented backbone provides in comparison to a notochord as is present in the North Atlantic hagfish. The hypothesis is that fish evolved from notochords to segmented backbones. In our research, we have focused on finding likely paths for the evolution of segmented backbones from notochords for which the segmented backbones provides an advantage in either navigation, speed, efficiency or some other combination of criteria. This paper describes the methodology that we use to test our predictions using both robotic (mechanical) and digital (virtual)

life forms. The methodology is based on the use and evaluation of robotic and digital organisms (animats) while they emulate or simulate swimming motion. The experiments help us to determine what advantages backbones provide for swimming in cartilaginous fish.

II. PROBLEM: DETERMINING HOW BACKBONES EVOLVED IN FISH

In spite of their name, vertebrates first evolved without vertebrae. Both phylogenetic reconstruction and fossil record agree that the hypothetical vertebrate ancestor possessed a continuous, unsegmented notochord as its axial skeleton [1], [2], [3], [4]. Only in jawed fishes did vertebral centra appear, gradually and repeatedly forming as segmental ossifications of perichordal tissue and/or cylindrical walls of the collagenous notochord sheath. Ring centra created intervertebral joints by leaving unmineralized connective tissues between adjacent body elements. Thus, the evolution of vertebral centra can be viewed as creation of intervertebral joints. Our primary objective is to test predictions about mechanical and evolutionary consequences of centra. Our primary mechanical prediction is that centra increase the axial skeleton's flexural stiffness by concentrating total curvature at joints. Our primary evolutionary prediction, based on the phylogenetic pattern of convergence is that centra have evolved in response to similar selection pressures; specifically, because of differences in swimming behavior between notochordal and vertebrated fishes [5], selection has been for higher velocity, acceleration, and versatility [6].

III. RELATED WORK

There has been much work based on using artificial life forms to study and emulate behavior and motion in biological systems. Terzopoulos et al [25] created a virtual oceanarium filled with artificial fish. They were primarily interested in emulating the behaviors of schools of fish. More recently, Ijspeert et al [26], [15] have used neural nets to emulate the central pattern generator in controlling swimming and walking motions.

There is also a large body of work that is more closely linked to studying the swimming motion of fish. There have been several pieces of research studying the swimming motion of fish under various conditions [27], [28], [29], [30], [31], [32], [33].

The models have becoming increasingly complex as we more fully understand the bio-mechanics underlying swimming.

The existing research in biology has mostly dealt with either (1) understanding the mechanics underlying swimming or (2) trying to create control structures that result in behaviors that are similar to that observed in biological systems. Work on evolution has been concentrated more on identifying differentiating features and determining the advantages that the features provide. It is much more difficult to determine the paths of evolution because of the absence of fossil records.

IV. METHODOLOGY

Our primary methodology is biomimetic evolutionary analysis, extending the field of ALife by creating biologically-based simulations (robotic and digital animats, [7]) of fish with phenotypically-variable axial skeletons swimming, navigating, and competing in artificial selection environments. A key innovation enabling use of robots is a biomaterials method to produce artificial collagen hydrogel structures that mimic properties of real notochords [8], [9]; to notochords we add artificial vertebral elements [10]. Digital animats combine physiologically-accurate models of the internal mechanics of fish [11] with an innovative computational implementation of the dynamics of the water surrounding the swimmer [12]. Phenotypically-variable animats, compete in a locomotor task, earn a fitness score, reproduce sexually at a level determined by fitness and in combination with random mutation, and then produce unique offspring. Figure 1 shows the historical pattern of backbones in fish species. The solid black lines indicate the species with solid backbones, the grayed lines show species with compound, hollow or partial backbones while the white bars show species with notochords. In spite of the historical pattern of convergence, the outcomes of our evolutionary experiments are anything but certain: even though robotic and digital worlds reward improved locomotion, differences exist in the animats and the specifics of their selection environments. Different selection environments are likely to yield different evolutionary trajectories [13], [14].

Robotic and digital animats differ significantly in generation time and population size: by virtue of the need to construct and test physical components, evolution of robotic animats progresses slowly. Thus for robotic animats we apply a methodology based on a modified form of genetic algorithms [15] that uses a small number of individuals in the population. For our digital animats we are able to push beyond traditional GA methodologies as an integral part of this project, we are developing and refining a GA-based optimization suite that has proven effective in identifying optimal morphological configurations for biting snakes [16], [17] and subsequently for swimming fish. Our innovation is to formulate parameter value selection as an optimization problem in and of itself, where the goal, here, is to minimize differences between the locomotor kinematics of the digital animat and its biological referent. As an intermediate step, we minimize the differences between the digital and robotic animats and subsequently the differences between the robotic animat and its biological referent. This two step process allows us to focus on the mechanical structures used to support swimming.

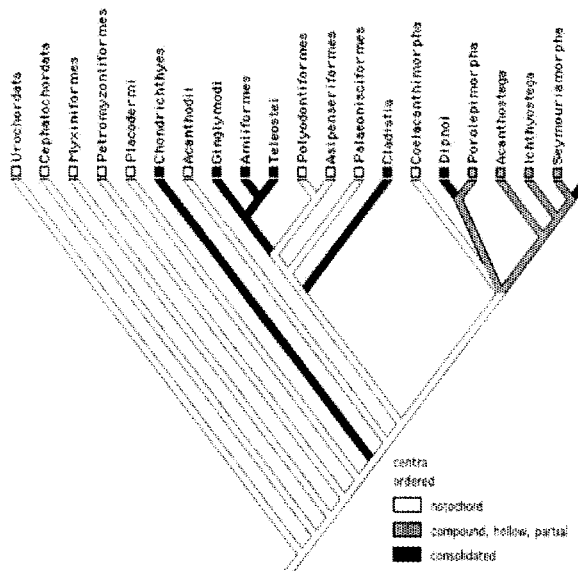


Fig. 1. Fish Phylogeny

Our two populations of animats permit, by analogy, immigration: phenotypes can be transferred and translated from digital to robotic environments and vice versa. To our knowledge, our methodology is unique in evolving digital and robotic animats in parallel with feedback between the two populations to improve them simultaneously.

A. Robotic Animat

We started with a fish-like robotic animat (Tadro) (Figure 2) that detects a light source, navigates towards it, and holds station in orbits around it [18], [19]. The robotic animat is biologically-inspired, employing a single eyespot linked to a tail offset mechanism that sea squirt tadpole larvae (*Chordata*, *Urochordata*) use for phototaxis [20]. We model navigation since it is fundamental for survival, a basic behavior for all autonomous agents (Meyer, 1997; Nelson et al. 2004).

Figure 2 shows the robotic animat. It consists of a roughly cylindrical plastic basin with a keel and a tail for propulsion. The basin holds a light-sensing "eye" that is off the axis of symmetry, a servo motor that oscillates the tail, and a simple control device that governs the attitude of the tail based on the intensity of light. The tail is attached to the servo motor via a vertical drive shaft. The tadro is intended to float with the contents of the basin exposed to air and the tail, keel, and the lateral walls of the basin submerged. The tail (Figure 3) is designed to mimic the caudal portion of a fish with a distinct caudal fin, provide dorso-ventral support for the integrated axial skeleton, and permit lateral bending influenced by the mechanical properties of the integrated axial skeleton. No muscles are present to modify the traveling flexural wave as in real fish [21]. The lateral offset

$$\begin{aligned} \dot{u} = & \frac{1}{M} \left\{ T \cos(\phi) - C_{D1} R L_D V u - \right. \\ & + \frac{A_T}{16} C_N(\gamma) \left[(4v - L_T \omega \cos \theta - 4R\omega \cos \phi - \right. \\ & L_T \omega \cos(\beta + \phi)) \times \\ & (2v \cos \phi - 2u \sin \phi - 2R\omega - L_T \omega \cos \beta) - \\ & \left. 4(u \cos \phi + v \sin \phi)(2u + \omega(2R + L_T \cos \beta) \sin \phi) \right] + \\ & \left. \frac{A_K}{2} C_N(\gamma') \left[(v^2 - u^2) \cos \theta - 2uv \sin \theta \right] \right\} \quad (4) \end{aligned}$$

$$\begin{aligned} \dot{v} = & \frac{1}{M} \left\{ T \sin(\phi) - C_{D1} R L_D V v - \right. \\ & \frac{A_T}{16} C_N(\gamma) \left[(4u + L_T \omega \sin \theta + 4R\omega \sin \phi + \right. \\ & L_T \omega \sin(\beta + \phi)) \times \\ & (2u \sin \phi - 2v \cos \phi + 2R\omega + L_T \omega \cos \beta) - \\ & \left. - 4(u \cos \phi + v \sin \phi)(2v - \omega(2R + L_T \cos \beta) \cos \phi) \right] + \\ & \left. \frac{A_K}{2} C_N(\gamma') \left[-2uv \cos \theta + (u^2 - v^2) \sin \theta \right] \right\}, \quad (5) \end{aligned}$$

$$\begin{aligned} \dot{\omega} = & \frac{1}{m\omega I} \left\{ -TR \sin \beta - 0.006\pi R^4 (L_D + \frac{1}{4}) |\omega| \omega - \right. \\ & \frac{A_T C_N(\gamma)}{16\sqrt{2}V} (2R + L_T \cos \beta) [\omega(2R + L_T \cos \beta) \times \\ & (u \cos \theta + v \sin \theta) + 2V^2 \sin \beta] \times \\ & \left[8V^2 + (2L_T^2 \cos^2 \beta + 8L_T R \cos \beta + 8R^2) \omega^2 + \right. \\ & (4L_T [u(\sin \theta + \sin(2\beta + \theta)) + \\ & v(\cos \theta + \cos(2\beta + \theta))]) + \\ & \left. 16R(u \sin(\beta + \theta) - v \cos(\beta + \theta)) \omega \right]^{\frac{3}{2}} + \\ & \left. \frac{0.007 D_K}{8} [R^4 \omega^2 + 4R^2 \omega (u \sin \theta - v \cos \theta) + 4V^2] \right\} \quad (6) \end{aligned}$$

where we make the definitions $V = \sqrt{u^2 + v^2}$, $\phi = \theta + \beta$ and

$$T = \begin{cases} \frac{3}{4} \pi^2 L_T f F (L_T - F \frac{3\pi}{2V}) & \text{if } V > F \frac{3\pi}{2L_T}, \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

The function $C_N(\gamma)$ can be expressed in terms of the parameters and variables of the problem as

$$C_N(\gamma) = \left| \frac{2\pi(v \cos \phi - u \sin \phi - R\omega - \frac{1}{2} L_T \omega \cos \beta)}{u \cos \phi + v \sin \phi} \right| \quad (8)$$

if

$$\left| v \cos \phi - u \sin \phi - R\omega - \frac{1}{2} L_T \omega \cos \beta \right| < \sin(\pi/18) V \quad (9)$$

and

$$C_N(\gamma) = (0.222 + 0.283 \frac{V}{|v \cos \phi - u \sin \phi - R\omega - \frac{1}{2} L_T \omega \cos \beta|})^{-1} \quad (10)$$

In a similar fashion, we can express $C_N(\gamma')$ as

$$C_N(\gamma') = \left| \frac{2\pi(u \cos \theta + v \sin \theta)}{v \cos \theta - u \sin \theta} \right| \quad (11)$$

$$|u \cos \theta + v \sin \theta| < \sin(\pi/18) V \quad (12)$$

$$C_N(\gamma') = \left(0.222 + 0.283 \frac{V}{|u \cos \theta + v \sin \theta|} \right)^{-1} \quad (13)$$

otherwise

V. EXPERIMENTS WITH ANIMATS

The goal of our research is to validate the hypothesis that backbones convey an advantage over notochords. Our methodology is based on using digital animats, based on robotic animats that are modeled after biological tadpoles combined with an optimizer to find the best type of tail for a specific body type (configuration) and environment. The parameter values to be used with the digital animats are determined experimentally by taking measurements with the robotic animats. A GA based optimizer [17] is then used to find the best type of tail for a specific environment (fitness function) through simulation. The values obtained for the best tail from these computer simulations are then used to construct the equivalent artificial tail for the robotic animats. Further experiments are then conducted with the robotic animats to validate the results from the computer simulations, i.e. the new tail provides the best performance for the specified environment. If there are discrepancies, it means that the computer simulation does not accurately reflect the robotic animats and their environment. These discrepancies are resolved through changes in the model of both the digital animat and its environment or the fitness function.

We formulate the problem of finding the best tail configuration (length, stiffness) as an optimization problem. This problem has a large number of parameters (between 7 and 12 variables) and so we used a GA based optimizer that has been shown to be successful on problems in high dimensional spaces and that was also able to generate results with relatively few evaluations. Our approach has several advantages:

- *ability to explore a large search space:*

The use of digital animats allows us to explore and evaluate many more tail configurations than would be possible with the robotic animats. The robotic animats and artificial tails are each time consuming to construct. The artificial tails can vary widely in length and stiffness, i.e., there are many possible combinations of both. Simulation with the digital animats allows us to explore the performance of many configurations before building an artificial tail and evaluating it on a robotic animat.

- *validation of computer models:*

The robotic animats are used to validate the models used to construct the digital robots, i.e., to evaluate whether all the important physical constraints have been considered. It is fairly easy to construct digital robots that can exhibit the desired behavior without using mechanisms that can be physically realized, i.e., in a mechanical robot. For example, the earliest digital animat was just a sphere with a tail, i.e., without a keel. The experiments with the animat showed that it was able to maintain station around the light source. In depth examination of the data revealed that the animat was also rotating around its own axis while circling the light source, clearly an undesirable result. Comparison of the experimental data led us to add a keel to the digital model, making it more complex but more realistic. In addition, further experiments that our simulation of swimming motion is incomplete if we do not model the forces interacting with the surrounding fluid. We are in the process of extending the model although it greatly increases the computational requirements, by as much as a factor of 100.

- *exploration of evolutionary paths:*

Evaluation of the animats (digital and robotic) allows us to more easily determine if the different points on our hypothesized evolution trajectory have similar performance characteristics as to what we believe. If the experimental data hold, then we have discovered one possible evolutionary path. There are many possible paths and at the moment, we are unable to discriminate between them to determine the likelihood of any of them. The results of our experiments are an improvement over the current state where many paths are postulated but there is no evidence to support any of them. The approach helps us find evidence to discriminate between the paths that have supporting evidence and those that do not.

A. Experimental Evaluation

This section describes the experimental setup for both the robotic and digital animats. The goal is to make the two environments as close to identical as possible. There are several major differences that arise due to differences in construction and modeling.

A.1 Experiments With The Robotic Animat

The behavioral goal for the animats is to navigate in a vector gradient (light) to a spot of high intensity in a water-filled tank. This environment mimics a situation where individuals compete in a navigational task to find a resource. Light is the proxy for an odorant plume, which, in hagfish, is known to provide sufficient information about food location for navigation [23]. The goal of the experiments was to determine for the robotic and digital animats, the best tail configuration for station keeping, i.e., staying in proximity to the light source.

For our initial experiments with robotic animats, three animats with artificial tails are placed in a 3x4m tank. In a given generation of fixed axial phenotypes, the three robotic animats compete and interact head-to-head. Tails are rotated among the

robots to avoid bias caused by physical differences among machines, and the starting positions of robotic animats are randomized to avoid systematic bias. To assess relative fitness among the animats of a given generation, we measure the following in each trial:

1. t , time to source: measure of navigational speed
2. p , pathlength to source: measure of navigational work
3. e , energy usage: measure of efficiency
4. r , radius of first station-holding orbit: related to turning abilities).

Trials are recorded and analyzed using videotape (for t , p , and r) and the power draw of each battery is logged continuously by the on-board microcontroller (for e). The details of these earlier experiments are described in [24].

A.2 Experiments With The Digital Animat

The digital animat was implemented using the equations described previously in section IV, (Equation 1 through Equation 6). The initial difficulty lay in finding "reasonable values" to assign to many of the parameters, e.g., intensity and placement of the light source as well as the appropriate Reynolds numbers to represent the fluid regime of the animats. A combination of simulations using both the robotic and digital animats were used to obtain the values. The light was modeled as a point source at a height of 2m above the pool to more accurately represent the environment of the robotic animat. Note that a light source above the pool has very different characteristics than a light source placed in the pool in the same plane as the animat. Specifically, the light intensity degrades more gradually and smoothly if the light source is placed above the pool.

The final experimental configuration is an optimization problem with the following seven (7) variables:

1. *flexural stiffness of the tail*: a stiffer tail is more powerful but less maneuverable
2. *length of the tail*: a longer tail is more powerful but less maneuverable
3. *frequency of the tail beat*: a higher frequency results in a higher velocity but a larger turning radius (less maneuverable)
4. *initial conditions*: the initial trajectory is composed of
 - speed in the x direction
 - speed in the y direction
 - angular velocity
 - orientation of the body

The initial conditions vary with the configuration. Given a fixed starting point, a less maneuverable fish is likely to overshoot (miss) the light source unless its initial velocity and orientation have values so that the animat can have time to change its course and make appropriate adjustments.

The values that we used for the digital animat are shown in Table I.

We used an optimizer that had been shown to work well for problems in high dimensional spaces [17] since our intent is to increase the number of variables in the digital model as time

TABLE I
PARAMETER VALUES OF THE DIGITAL ANIMAT

Parameter	Value
mass of the body & tail	0.857 kg
moment of inertia	3.122 kg/m ²
radius of the body	0.025 m
depth of the body (underwater)	0.051 m
depth of the keel	0.1 m
area of the keel	0.02 m ²

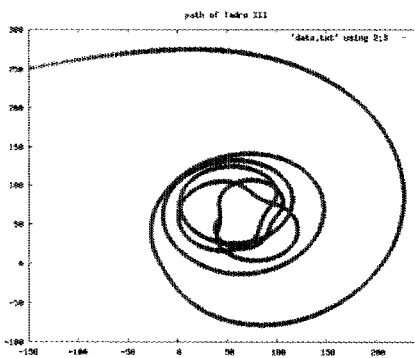


Fig. 5. Path of Digital Animat

goes on. As the animats and their environments (fitness functions) become increasingly more complex, the number of controlling parameters increases. This typically has the effect of exponentially increasing the computational time of each simulation. The current experimental results were obtained with a simple fitness function that measured the overall sum of the distances of the animat from the light source over a fixed time period. This measurement incorporates both (1) the speed of the animat and (2) its station keeping ability, i.e., to circle around a fixed point. Figure 5 shows one path of the digital animat as it orients and swims around a light source hanging overhead at (0,0). Note that it is not able to maintain a path centered around (0,0) but is able to generally swim around the light.

The simulations showed that extremas were undesirable, i.e., extremely long or short tails and extremely stiff or flexible tails for station keeping. However, the experiments with the digital animats do not quite correspond with those of the robotic animats because the fitness function used for the digital animats is not the same as was used for the robotic animats. There are several criteria used for the robotic animats that could not be measured using the digital animats. Primarily, we could not measure

swimming efficiency because there was no way to determine the amount of effort used to move the tail for a specified frequency and amplitude. In the robotic animat, this is measured by observing the current used by the servo motor to drive the tail. An equivalent system in the digital animat would require that we model the muscles used to move the tail.

We are currently using the results of the experiments to construct a new artificial tail that will then be evaluated using the robotic animats. The results of those experiments will show us whether the differences in fitness functions have a perceptible effect. We are also carrying out additional experiments to see how the stability point, i.e., the optimum combination of length and stiffness, is affected by the values of the other parameters in the simulation.

VI. FUTURE WORK

There is a need for much more work to be done on understanding evolution and the paths along which species have evolved. This paper has described a methodology that can be used to determine likely evolutionary paths for the evolution of segmented backbones in swimming fish (*carangiform* motion). Our results would be much improved if the robotic and digital fish more closely resembled fish both physically and in behavior. There are several areas of the digital fish where we are extending our efforts:

realistic fish models: We have developed a much more complex mathematical model of fish locomotion based on studies of the pumpkinfish. [12] The model incorporates fluid forces and will eventually include neuro-muscular activation mechanisms. The cost of this model is that it is much more expensive computationally to run. The current digital model takes approximately 3 hours to analyze while the more complex model requires approximately 20 days. This makes it infeasible for quick studies even though the time frame is still shorter than that required to construct new artificial tails.

fitness functions: We currently use a simple fitness function that only represents station keeping, i.e., the ability to maintain a swimming orbit around a fixed point, e.g., a light source. More complex fitness functions should include other factors such as prey avoidance and swimming efficiency. These criteria require more powerful simulation models and environments to be developed and also are more difficult to construct because of the difficulty in quantifying the criteria. There is also the additional problem of determining how to combine the factors into a single function, and also the weight that should be applied to each. The factors are not completely independent of one another even though they are treated as such at this time.

schooling behaviors: The current simulation only models the swimming motion of a single fish. It would be helpful to be able to model an environment with multiple fish, either as a school or in a predator/prey context.

VII. CONCLUSION

This paper has described a new methodology for using artificial life systems to help understand the workings of biological systems. Specifically, we construct robotic and digital animats

to simulate the mechanics underlying swimming motion in fish. Experiments with the animats have been used to evaluate our hypothesis that one possible reason for the evolution of backbones in fish is that they provide additional strength and stiffness which in turn results in higher velocity, acceleration and maneuverability.

The combination of digital and robotic animats provides the ability to (1) search very large spaces using computer simulations and (2) verify the physics and mathematics principles underlying the digital animat and its environment through emulations with the robotic animat. Without the combination of animats, we would either be searching a very small part of the space or be constructing animats that are not physically realizable.

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